Po definiciji odrediti izvod funkcije $y = \ln x$

$$y = \ln x$$

$$y' = \lim_{x \to 0} \frac{\ln(x + \Delta x) - \ln x}{\Delta x}$$

$$y' = \lim_{x \to 0} \frac{\ln \frac{(x + \Delta x)}{x}}{\Delta x}$$

$$y' = \lim_{x \to 0} \frac{1}{\Delta x} \ln \frac{x + \Delta x}{x}$$

$$y' = \lim_{x \to 0} \ln \left(\frac{x + \Delta x}{x} \right)^{\frac{1}{\Delta x}}$$

$$y' = \lim_{x \to 0} \ln \left(1 + \frac{\Delta x}{x} \right)^{\frac{1}{\Delta x}}$$

Uvodimo smenu:

$$\frac{\Delta x}{x} = u$$

$$\frac{1}{\Delta x} = \frac{1}{ux}$$

$$x \to 0 \Rightarrow u \to 0$$

$$y' = \lim_{x \to 0} \ln \left(1 + \frac{\Delta x}{x} \right)^{\frac{1}{\Delta x}}$$

$$y' = \lim_{u \to 0} \ln (1 + u)^{\frac{1}{ux}}$$

$$y' = \lim_{u \to 0} \ln \left((1 + u)^{\frac{1}{u}} \right)^{\frac{1}{x}}$$

$$y' = \lim_{u \to 0} \frac{1}{x} \ln (1 + u)^{\frac{1}{u}}$$

$$y' = \frac{1}{x}\ln\lim_{u\to 0}(1+u)^{\frac{1}{u}}$$

$$y' = \frac{1}{x}\ln e$$

$$y' = \frac{1}{x}$$