

519. Ako je $\log_b x = \frac{1}{2}(\log_a x + \log_c x)$, dokazati da je

$$\log_b \sqrt{ac} = \log_b a \cdot \log_b c \quad (a, b, c, x > 0, a, b, c, x \neq 1)$$

$$\log_b x = \frac{1}{2}(\log_a x + \log_c x)$$

$$\log_b x = \frac{1}{2} \left(\frac{\log_b x}{\log_b a} + \frac{\log_b x}{\log_b c} \right), \text{ podelimo levu i desnu stranu jednačine sa } \log_b x$$

$$1 = \frac{1}{2} \left(\frac{1}{\log_b a} + \frac{1}{\log_b c} \right)$$

$$1 = \frac{1}{2} \frac{\log_b a + \log_b c}{\log_b a \cdot \log_b c}$$

$$2 \cdot \log_b a \cdot \log_b c = \log_b a + \log_b c, \text{ podelimo levu i desnu stranu jednačine sa } 2$$

$$\frac{1}{2}(\log_b a + \log_b c) = \log_b a \cdot \log_b c$$

$$\frac{1}{2} \log_b ac = \log_b a \cdot \log_b c$$

$$\log_b (ac)^{\frac{1}{2}} = \log_b a \cdot \log_b c$$

$$\log_b \sqrt{ac} = \log_b a \cdot \log_b c$$