403. Dokazati da je $\left(\sqrt{2}+i\right)^6+\left(\sqrt{2}-i\right)^6=-46$.

Prvo treba uočiti sledeće:

$\left(\sqrt{2}+i\right)^2=1+2\sqrt{2}i$, kao i da je $\left(\sqrt{2}-i\right)^2=1-2\sqrt{2}i$

$$\left(\sqrt{2}+i\right)^6+\left(\sqrt{2}-i\right)^6=-46$$

$$\left(\left(\sqrt{2}+i\right)^2\right)^3+\left(\left(\sqrt{2}-i\right)^2\right)^3=-46$$

$$\left(1+2\sqrt{2}i\right)^3+\left(1-2\sqrt{2}i\right)^3=-46$$

Primenićemo ovde jedan mali trik:

$$(a+b)^3+(a-b)^3=a^3+3a^2+3ab^2+b^3+a^3-3a^2+3ab^2-b^3$$
$$(a+b)^3+(a-b)^3=2a^3+6ab^2 \qquad (1)$$

Ako imamo:

$$\left(1+2\sqrt{2}i\right)^3+\left(1-2\sqrt{2}i\right)^3=-46$$

U skladu sa jednačinom (1):

$$\left(1+2\sqrt{2}i\right)^3+\left(1-2\sqrt{2}i\right)^3=2\cdot 1^3+6\cdot 1\cdot\left(2\sqrt{2}i\right)^2$$

$$\left(1+2\sqrt{2}i\right)^3+\left(1-2\sqrt{2}i\right)^3=2+6\cdot(-8)$$

$$\left(1+2\sqrt{2}i\right)^3+\left(1-2\sqrt{2}i\right)^3=-46$$

$$\left(\sqrt{2}+i\right)^6+\left(\sqrt{2}-i\right)^6=-46$$