

1080 d. Dokazati da za sve prirodne brojeve n važi:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Prvo pokazujemo da važi za:

$$\text{za } n = 1, \quad 1^2 = \frac{1 \cdot (1+1)(2+1)}{6}$$

$$\text{za } n = 2, \quad 1^2 + 2^2 = \frac{2 \cdot (2+1)(4+1)}{6}$$

$$\text{za } n = 3, \quad 1^2 + 2^2 + 3^2 = \frac{3 \cdot (3+1)(6+1)}{6}$$

Pretpostavimo da važi i za $n = k$, INDUKCIJSKA HIPOTEZA (IH)

$$1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k \cdot (k+1)(2k+1)}{6}$$

Tada je za $n = k + 1$, INDUKCIJSKI KORAK (IK)

$$1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1) \cdot [(k+1)+1] \cdot [2(k+1)+1]}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

‘Pozivamo’ indukcijsku hipotezu:

$$\frac{k \cdot (k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{k \cdot (k+1)(2k+1)}{6} + \frac{6 \cdot (k+1)^2}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{k \cdot (k+1)(2k+1) + 6 \cdot (k+1)^2}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)[(2k^2+k)+6(k+1)]}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)[2k^2+k+6k+6]}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)(2k^2+4k+3k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)[2k(k+2)+3(k+2)]}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

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