107. a) Odrediti funkcije $f(x)$ i $g(x)$, koje zadovoljavaju sisteme:

$$f(2x+1)+g(x-1)=x \quad \text{i} \quad f(2x+1)-2g(x-1)=2x^2$$

$$f(2x+1)+g(x-1)=x \qquad\qquad (1)$$
$$f(2x+1)-2g(x-1)=2x^2 \qquad (2)$$

Nakon oduzimanja druge jednačine od prve, dobijamo:

$$3g(x-1)=x-2x^2$$
$$g(x-1)=\frac{x-2x^2}{3}, \quad \text{uvodimo smenu}$$

$$x-1=t$$
$$x=t+1$$

$$g(t)=\frac{t+1-2\cdot(t+1)^2}{3}$$
$$g(t)=\frac{t+1-2\cdot(t^2+2t+1)}{3}$$
$$g(t)=\frac{t+1-2t^2-4t-2}{3}$$
$$g(t)=\frac{-2t^2-3t-1}{3}$$
$$g(t)=-\frac{1}{3}\cdot(2t^2+3t+1)$$
$$g(x)=-\frac{1}{3}\cdot(2x^2+3x+1)$$

Kako je , $g(x-1)=\dfrac{x-2x^2}{3}$ to vraćamo u jednačinu (1) $f(2x+1)+g(x-1)=x$

$$f(2x+1)+\frac{x-2x^2}{3}=x$$
$$f(2x+1)=\frac{3x-x+2x^2}{3}$$

$$f(2x+1) = \frac{2x + 2x^2}{3} \ , \ \text{uvodimo smenu :}$$

$$2x + 1 = t$$
$$2x = t - 1$$
$$x = \frac{t-1}{2}$$

Kako je $\quad f(2x+1) = \frac{2}{3} \cdot (x + x^2)$

$$f(t) = \frac{2}{3} \cdot \left(\frac{t-1}{2} + \left(\frac{t-1}{2} \right)^2 \right)$$

$$f(t) = \frac{2}{3} \cdot \frac{2(t-1) + t^2 - 2t + 1}{4}$$

$$f(t) = \frac{1}{6} \cdot (2t - 2 + t^2 - 2t + 1)$$

$$f(t) = \frac{1}{6} \cdot (t^2 - 1)$$

$$f(x) = \frac{1}{6} \cdot (x^2 - 1)$$

Rešenja su :

$$f(x) = \frac{1}{6} \cdot (x^2 - 1) \quad \text{i} \quad g(x) = -\frac{1}{3} \cdot (2x^2 + 3x + 1)$$