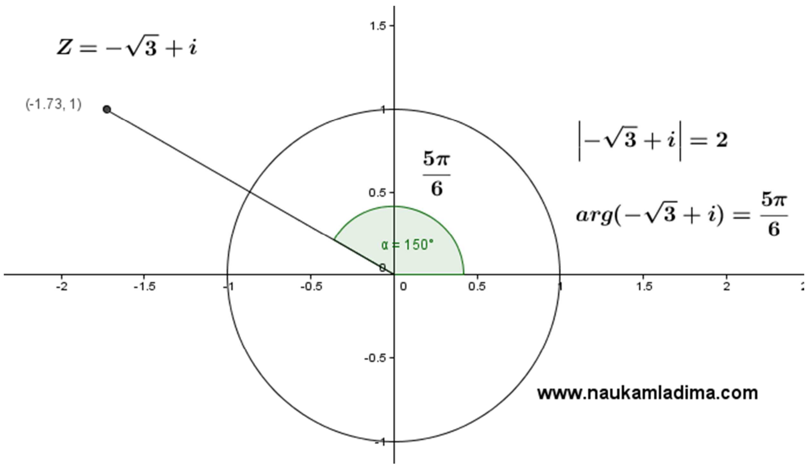
8. Naći sve šeste korene broja $i-\sqrt{3}$ .

Kompleksni broj $i-\sqrt{3}$ , odnosno $-\sqrt{3}+i$ možemo predstaviti u kompleksnoj ravni.




$$|i-\sqrt{3}|=2\,,\ arg(i-\sqrt{3})=\frac{5\pi}{6}$$

$$i-\sqrt{3}=2\left(cos\frac{5\pi}{6}+i\,sin\frac{5\pi}{6}\right),\quad \frac{5\pi}{6}\in(-\pi,\pi]$$

Zadatak je da se izračuna $\sqrt[6]{i-\sqrt{3}}$

$$Z_k == \sqrt[6]{2}\left(cos\frac{\frac{5\pi}{6}+2k\pi}{6}+i\,sin\frac{\frac{5\pi}{6}+2k\pi}{6}\right)$$

$$k\in\{0,1,2,3,4,5\}$$

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Za k= 0:

$$Z_0=\sqrt[6]{2}\left(cos\frac{\frac{5\pi}{6}}{6}+i\,sin\frac{\frac{5\pi}{6}}{6}\right)$$

$$Z_0=\sqrt[6]{2}\left(cos\frac{5\pi}{36}+i\,sin\frac{5\pi}{36}\right)\ ,\quad \frac{5\pi}{36}=25°$$

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Za k= 1:

$$Z_1=\sqrt[6]{2}\left(cos\frac{\frac{5\pi}{6}+2\pi}{6}+i\,sin\frac{\frac{5\pi}{6}+2\pi}{6}\right)$$

$$Z_1=\sqrt[6]{2}\left(cos\frac{17\pi}{36}+i\,sin\frac{17\pi}{36}\right)\ ,\frac{5\pi}{36}=85°$$

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Za k= 2:

$$Z_2 = \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{6}+4\pi}{6} + i\sin\frac{\frac{5\pi}{6}+4\pi}{6}\right)$$

$$Z_2 = \sqrt[6]{2}\left(\cos\frac{29\pi}{36} + i\sin\frac{29\pi}{36}\right), \ \frac{29\pi}{36} = 145°$$

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Za k= 3:

$$Z_3 = \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{6}+6\pi}{6} + i\sin\frac{\frac{5\pi}{6}+6\pi}{6}\right)$$

$$Z_3 = \sqrt[6]{2}\left(\cos\frac{41\pi}{36} + i\sin\frac{41\pi}{36}\right), \ \frac{41\pi}{36} = 205°$$

$$\frac{41\pi}{36} - 2\pi = \frac{41\pi}{36} - \frac{72\pi}{36} = -\frac{31\pi}{36}, \quad -\frac{31\pi}{36} \in (-\pi, \pi]$$

$$Z_3 = \sqrt[6]{2}\left(\cos\left(-\frac{41\pi}{36}\right) + i\sin\left(-\frac{41\pi}{36}\right)\right), -\frac{31\pi}{36} = -155°$$

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Za k= 4:

$$Z_4 = \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{6}+8\pi}{6} + i\sin\frac{\frac{5\pi}{6}+8\pi}{6}\right)$$

$$Z_4 = \sqrt[6]{2}\left(\cos\frac{53\pi}{36} + i\sin\frac{53\pi}{36}\right), \ \frac{53\pi}{36} = 265°$$

$$\frac{53\pi}{36} - 2\pi = \frac{53\pi}{36} - \frac{72\pi}{36} = -\frac{19\pi}{36}, \quad -\frac{19\pi}{36} \in (-\pi, \pi]$$

$$Z_4 = \sqrt[6]{2}\left(\cos\left(-\frac{19\pi}{36}\right) + i\sin\left(-\frac{19\pi}{36}\right)\right), -\frac{19\pi}{36} = -95°$$

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Za k= 5:

$$Z_5 = \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{6}+10\pi}{6} + i\sin\frac{\frac{5\pi}{6}+10\pi}{6}\right)$$

$$Z_5 = \sqrt[6]{2}\left(\cos\frac{65\pi}{36} + i\sin\frac{65\pi}{36}\right), \ \frac{65\pi}{36} = 325°$$

$$\frac{65\pi}{36} - 2\pi = \frac{65\pi}{36} - \frac{72\pi}{36} = -\frac{7\pi}{36}, \quad -\frac{7\pi}{36} \in (-\pi, \pi]$$

$$Z_4 = \sqrt[6]{2}\left(cos\left(-\frac{7\pi}{36}\right) + i\, sin\left(-\frac{7\pi}{36}\right)\right), -\frac{7\pi}{36} = -35°$$

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Konačno, sva rešenja možemo da predstavimo u kompleksnoj ravni .