680. Iz $\frac{tg(x-y)}{tgx}+\frac{\sin^2 z}{\sin^2 x}=1$ sledi $tg^2 z = tgx\cdot tgy$. Dokazati.

$$\frac{tg(x-y)}{tgx}+\frac{\sin^2 z}{\sin^2 x}=1$$
$$\frac{\sin^2 z}{\sin^2 x}=1-\frac{tg(x-y)}{tgx}$$
$$\frac{\sin^2 z}{\sin^2 x}=1-\frac{\sin(x-y)\cdot\cos x}{\cos(x-y)\cdot\sin x}$$
$$\frac{\sin^2 z}{\sin^2 x}=\frac{\sin x\cdot\cos(x-y)-\sin(x-y)\cdot\cos x}{\cos(x-y)\cdot\sin x}$$

Kako je $\sin(x-y)=\sin x\cdot\cos y-\sin y\cdot\cos x$

$$\frac{\sin^2 z}{\sin^2 x}=\frac{\sin(x-(x-y))}{\cos(x-y)\cdot\sin x}$$
$$\frac{\sin^2 z}{\sin^2 x}=\frac{\sin y}{\cos(x-y)\cdot\sin x}$$
$$\sin^2 z=\frac{\sin y\cdot\sin^2 x}{\cos(x-y)\cdot\sin x}$$

Kako je $\sin^2 z=\frac{\sin x\cdot\sin y}{\cos(x-y)}$ tada je i $\cos^2 z=1-\frac{\sin x\cdot\sin y}{\cos(x-y)}$

Pošto je $tg^2 z=\frac{\sin^2 z}{\cos^2 z}$ onda je i :

$$tg^2 z=\frac{\frac{\sin x\cdot\sin y}{\cos(x-y)}}{1-\frac{\sin x\cdot\sin y}{\cos(x-y)}}$$

$$tg^2 z=\frac{\sin x\cdot\sin y}{\cos(x-y)-\sin x\cdot\sin y}$$ i kako je $\cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y$

$$tg^2 z=\frac{\sin x\cdot\sin y}{\cos x\cdot\cos y+\sin x\cdot\sin y-\sin x\cdot\sin y}$$
$$tg^2 z=\frac{\sin x\cdot\sin y}{\cos x\cdot\cos y}$$

$tg^2 z=tgx\cdot tgy$ što je trebalo i dokazati. Sami odredite uslove za x, y i z !!!