

1258. Reši jednačinu: $\log_x \sqrt{5} + \log_x 5x - 2.25 = (\log_x \sqrt{5})^2$

$$\begin{aligned}\log_x \sqrt{5} + \log_x 5x - 2.25 &= (\log_x \sqrt{5})^2 \\ \log_x \sqrt{5} + \log_x 5 + \log_x x - 2.25 &= (\log_x \sqrt{5})^2 \\ \log_x \sqrt{5} + \log_x \sqrt{5}^2 + 1 - 2.25 &= (\log_x \sqrt{5})^2 \\ \log_x \sqrt{5} + 2\log_x \sqrt{5} - 1.25 &= (\log_x \sqrt{5})^2 \\ 3\log_x \sqrt{5} - 1.25 &= (\log_x \sqrt{5})^2\end{aligned}$$

Nakon ovoga, uvodimo smenu: $\log_x \sqrt{5} = t$

$$\begin{aligned}3t - 1.25 &= t^2 \\ t^2 - 3t + 1.25 &= 0 \\ t_{1,2} &= \frac{3 \pm \sqrt{9 - 5}}{2} \\ t_{1,2} &= \frac{3 \pm 2}{2} \\ t_1 &= \frac{5}{2} \quad t_2 = \frac{1}{2}\end{aligned}$$

$$\log_x \sqrt{5} = \frac{5}{2} \qquad \log_x \sqrt{5} = \frac{1}{2}$$

$$\frac{1}{\log_{\sqrt{5}} x} = \frac{5}{2} \qquad \frac{1}{\log_{\sqrt{5}} x} = \frac{1}{2}$$

$$\log_{\sqrt{5}} x = \frac{2}{5} \qquad \log_{\sqrt{5}} x = 2$$

$$\begin{aligned}x &= \sqrt{5}^{\frac{2}{5}} & x &= \sqrt{5}^2 \\ x &= \sqrt[5]{5} & x &= 5\end{aligned}$$