

1083. Rešiti jednačinu: $4^{x+\sqrt{x^2-2}} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} = 6$.

$$\begin{aligned}4^{x+\sqrt{x^2-2}} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} &= 6 \\(2^2)^{x+\sqrt{x^2-2}} - 5 \cdot 2^{-1+x+\sqrt{x^2-2}} &= 6 \\(2^{x+\sqrt{x^2-2}})^2 - \frac{5}{2} \cdot 2^{x+\sqrt{x^2-2}} &= 6\end{aligned}$$

Uvodimo smenu: $t = 2^{x+\sqrt{x^2-2}}$

$$\begin{aligned}t^2 - \frac{5}{2} \cdot t &= 6 \\2t^2 - 5t - 12 &= 0 \\t_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\t_{1,2} &= \frac{5 \pm \sqrt{25 - 96}}{4} \\t_{1,2} &= \frac{5 \pm \sqrt{121}}{4} \\t_1 &= 4 \quad t_2 = -\frac{3}{2} \text{ (ovo ne može da bude rešenje)}\end{aligned}$$

Sada vratimo smenu:

$$\begin{aligned}4 &= 2^{x+\sqrt{x^2-2}} \\2^2 &= 2^{x+\sqrt{x^2-2}} \\2 &= x + \sqrt{x^2-2}, \quad x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty) \\\sqrt{x^2-2} &= 2 - x, \text{ nakon kvadriranja dobijamo:}\end{aligned}$$

$$\begin{aligned}x^2 - 2 &= 4 - 4x + x^2 \\4x &= 6 \\x &= \frac{3}{2}\end{aligned}$$