1081 b. Dokazati da za sve prirodne brojeve *n* važi:

$$\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+.....+\frac{n^2+n+1}{n\cdot(n+1)}=\frac{n\cdot(n+2)}{n+1}$$

za $n=1$,

$$\frac{3}{1\cdot 2}=\frac{1\cdot(1+2)}{1+1}, \qquad \frac{3}{2}=\frac{3}{2}$$

za $n=2$,

$$\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}=\frac{2\cdot(2+2)}{2+1}; \qquad \frac{3}{2}+\frac{7}{6}=\frac{8}{3}$$

Pretpostavka: $n=k$

$$\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+.....+\frac{k^2+k+1}{k\cdot(k+1)}=\frac{k\cdot(k+2)}{k+1}$$

za $n=k+1$

$$\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+.....+\frac{k^2+k+1}{k\cdot(k+1)}+\frac{(k+1)^2+(k+1)+1}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k\cdot(k+2)}{k+1}+\frac{(k+1)^2+(k+1)+1}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k\cdot(k+2)^2+(k+1)^2+(k+1)+1}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k\cdot(k^2+4k+4)^2+k^2+2k+1+k+2}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k^3+4k^2+4k+k^2+3k+3}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k^3+5k^2+7k+3}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$(k^3+5k^2+7k+3):(k+1)=k^2+4k+3$ (deljenje polinoma polinomom)

k^3+k^2

$4k^2+7k$

$4k^2+4k$

$3k+3$

$3k+3$

$$\frac{k^3+5k^2+7k+3}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{(k+1)\cdot(k^2+4k+3)}{(k+1)\cdot(k+2)}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{(k^2+4k+3)}{k+2}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k^2+k+3k+3}{k+2}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{k\cdot(k+1)+3\cdot(k+1)}{k+2}=\frac{(k+1)\cdot(k+3)}{k+2}$$

$$\frac{(k+1)\cdot(k+3)}{k+2}=\frac{(k+1)\cdot(k+3)}{k+2}$$