

1081 a. Dokazati da za sve prirodne brojeve n važi:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

za $n=1$,

$$\frac{1}{1 \cdot 2} = \frac{1}{1+1}$$

za $n=2$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} = \frac{2}{2+1}; \quad \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$$

za $n=3$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{3}{3+1}; \quad \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{3}{4}$$

Pretpostavka: $n = k$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{k \cdot (k+1)} = \frac{k}{k+1}$$

Za $n = k+1$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{k \cdot (k+1)} + \frac{1}{(k+1) \cdot (k+2)} = \frac{k+1}{k+2}$$

$$\frac{k}{k+1} + \frac{1}{(k+1) \cdot (k+2)} = \frac{k+1}{k+2}$$

$$\frac{k(k+2)+1}{(k+1) \cdot (k+2)} = \frac{k+1}{k+2}$$

$$\frac{k^2 + 2k + 1}{(k+1) \cdot (k+2)} = \frac{k+1}{k+2}$$

$$\frac{(k+1)^2}{(k+1) \cdot (k+2)} = \frac{k+1}{k+2}$$

$$\frac{k+1}{k+2} = \frac{k+1}{k+2}$$